PERIODIC HEATING OF A SYSTEM OF CYLINDERS AND DETERMINING THE HEAT-TRANSFER COEFFICIENT ON THIS BASIS

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Relations are derived for the amplitudes ratio and the phase shift angle of the gas temperature during periodic heating of a bundle of cylindrical rods.

One interesting problem in thermophysics is the heating of various kinds of packages in a stream of gas. We will analyze a special case in detail. Let us consider a thermally insulated channel containing a bundle of cylindrical rods and passing a gas whose temperature at the inlet varies periodically (Fig. 1). It will be assumed that the heat-transfer coefficient and the thermophysical properties are not functions of space coordinates and time, also that the gas velocity remains constant at any section of the package; the equivalent thermal conductivity of the package in the direction of gas flow will be considered negligibly low. The temperature of the rods and of the gas is described by the following equations:

$$\frac{\partial t}{\partial \xi} + \frac{1}{v} \cdot \frac{\partial t}{\partial \tau} = \frac{\alpha F}{WL}(t_{\text{M0}} - t), \quad 0 < \xi < L, \quad \tau > 0, \tag{1}$$

$$\frac{\partial t_{\text{M}}}{\partial \tau} = a\left(\frac{\partial^2 t_{\text{M}}}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial t_{\text{M}}}{\partial r}\right), \quad 0 < r < R, \quad \tau > 0 \tag{2}$$

and the boundary conditions

$$t(0, \tau) = A_1 \cos \omega\tau, \tag{3}$$

$$\left.\frac{\partial t_{\text{M}}}{\partial r}\right|_{r=0} = 0, \quad \left.\frac{\partial t_{\text{M}}}{\partial r}\right|_{r=R} = \frac{\alpha}{\lambda_{\text{M}}}(t - t_{\text{M0}}). \tag{4}$$

We will seek the periodic component of the solution (disregarding the initial temperature distribution) in the form:

$$t(\xi, \tau) = A_1 \exp(p\xi + i\omega\tau), \tag{5}$$

where p is a complex constant.

The periodic solution to the equation of heat conduction is well known and can be found in the technical literature; in [1], for instance, the temperature of a cylinder surface in a medium whose temperature varies according to (5) is described by the relation

$$t_{\text{M0}} = A_1 N \exp(p\xi + i\omega\tau),$$

where

$$N = \frac{I_0\left(\sqrt{\frac{i\omega}{a}}R\right)}{I_0\left(\sqrt{\frac{i\omega}{a}}R\right) + \frac{\lambda_{\text{M}}}{\alpha}\sqrt{\frac{i\omega}{a}}\, I_1\left(\sqrt{\frac{i\omega}{a}}R\right)}.$$

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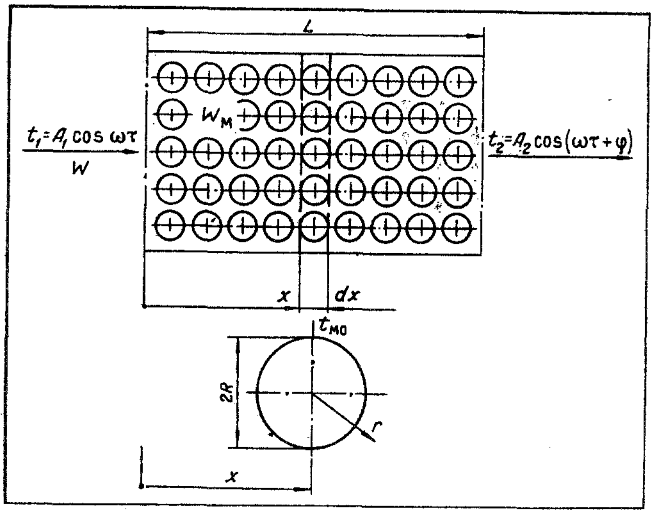

Fig. 1. Schematic diagram of the problem.

After inserting t and t_{MO} into Eq. (1), we obtain the following expression for the constant p:

$$p = \frac{\alpha F}{WL}(1-N) - \frac{i\omega}{v}.$$

Consequently, the gas temperature at the outlet will be

$$t(L, \tau) = A_2 \cos(\omega\tau + \varphi_c), \tag{6}$$

where the temperature amplitude at the outlet is determined from the ratio

$$\ln\frac{A_2}{A_1} = -\frac{\alpha F}{W}\,\mathrm{Re}\,(1-N).$$

For the total phase shift of temperature oscillations we have

$$\varphi_c = -\frac{\alpha F}{W}\,\mathrm{Im}\,(1-N) - \frac{\omega L}{v}.$$

With the quantity (1−N) resolved into it real and imaginary parts, the final formulas for the amplitude ratio and for the phase shift are

$$\ln\frac{A_1}{A_2} = \frac{\alpha F}{W}\cdot\frac{\frac{x}{\sqrt{2}}\,\mathrm{Bi}\,P_{0,1}(x) + 2x^2P_1(x)}{\mathrm{Bi}^2P_0(x) + x\sqrt{2}\,\mathrm{Bi}\,P_{0,1}(x) + x^2P_1(x)}, \tag{7}$$

$$\varphi_c = -\frac{\alpha F}{W}\cdot\frac{\frac{x}{\sqrt{2}}\,\mathrm{Bi}\,P_{1,0}(x)}{\mathrm{Bi}^2P_0(x) + x\sqrt{2}\,\mathrm{Bi}\,P_{0,1}(x) + x^2P_1(x)} - \frac{\omega L}{v}. \tag{8}$$

The functions of the argument $x = \sqrt{(\omega/a)}R$ are defined as follows:

$$P_0(x) = \mathrm{ber}_0^2 x + \mathrm{bei}_0^2 x, \quad P_1(x) = \mathrm{ber}_1^2 x + \mathrm{bei}_1^2 x,$$
$$P_{0,1}(x) = \mathrm{ber}_0 x(\mathrm{ber}_1 x + \mathrm{bei}_1 x) - \mathrm{bei}_0 x(\mathrm{ber}_1 x - \mathrm{bei}_1 x), \tag{9}$$
$$P_{1,0}(x) = -\mathrm{ber}_1 x(\mathrm{ber}_0 x + \mathrm{bei}_0 x) + \mathrm{bei}_1 x(\mathrm{ber}_0 x - \mathrm{bei}_0 x).$$

Some values of these functions for $x \in \langle 0, 10\rangle$ are listed in Table 1.

Let $H = \alpha F/W_M\omega$, then $x = \sqrt{2\mathrm{Bi}/H}$ and ratio (7) can be expressed as

$$Z = \frac{W}{W_M\omega}\ln\frac{A_1}{A_2} = Hf(H)\,\mathrm{Bi}. \tag{10}$$

TABLE 1. Some Values of Functions $P_0(x)$, $P_1(x)$, $P_{0,1}(x)$, and $P_{1,0}(x)$ Defined According to Formulas (9)

x	$P_0(x)$	$P_1(x)$	$P_{0,1}(x)$	$P_{1,0}(x)$
0,0	1,000	0,000	0,000	0,000
0,1	1,000	0,002	0,000	0,070
0,2	1,000	0,011	0,001	0,141
0,4	1,001	0,040	0,006	0,283
0,6	1,004	0,090	0,019	0,424
0,8	1,014	0,160	0,045	0,568
1,0	1,032	0,252	0,089	0,714
1,5	1,161	0,578	0,303	1,117
2,0	1,511	1,086	0,736	1,652
2,5	2,283	1,890	1,520	2,507
3,0	3,809	3,240	2,900	4,025
4,0	11,83	10,08	9,780	11,95
5,0	38,86	33,80	33,18	38,97
6,0	132,3	118,0	116,3	132,7
8,0	1688	1529	1515	1670
10,0	22470	20930	20830	22450

Relation (10) has been plotted in Fig. 2. The ratio of inlet to outlet temperature amplitudes is here a function of the dimensionless variables $\alpha F/W$, H, and Bi.

If the thermal conductivity of the package material is high, then this problem can be solved by letting $Bi \to 0$ in (7) and (8). We then obtain

$$\ln\frac{A_1}{A_2} = \frac{\alpha F}{W}\cdot\frac{1}{1+H^2}, \tag{11}$$

$$\varphi_c = -\frac{\alpha F}{W}\cdot\frac{H}{1+H^2} - \frac{\omega L}{v}, \tag{12}$$

from where

$$Z = \frac{H}{1+H^2}. \tag{13}$$

These formulas are valid regardless of the geometry of the package elements and they may be used instead of (7), (8), and (10), if $H < 0.1$ and $Bi < 0.2$.

The solution to such problems not only for a bundle of cylinders but also for a bed of spheres, a stack of plates, thin-walled channels, etc., is the basis for a new experimental method of determining the mean coefficients of convective heat transfer during a gas flow through various types of packages. An advantage of this so-called cyclic method is that the surface temperature of the package elements does not have to be measured. For instance, at a gas inlet temperature according to (3), one must only determine the inlet and the outlet temperature amplitudes and, at a given geometry and under known flow conditions,

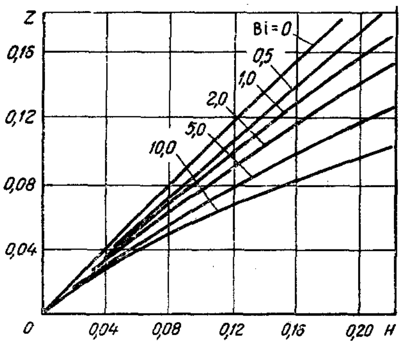

Fig. 2

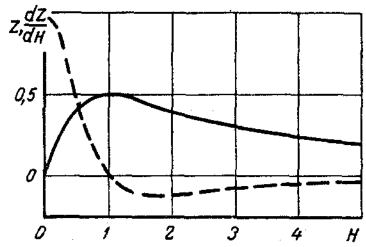

Fig. 3

Fig. 2. Function Z = f(H, Bi) for a package of cylinders.

Fig. 3. Function Z = f(H), dZ/dH = f'(H) for a package with $\lambda_M = \infty$: solid curve) Z; dashed curve) dZ/dH.

one obtains the heat-transfer coefficient from either H or the Bi number through Eqs. (7) or (11), respectively. For the same purpose, one may also use the fundamental component of the phase shift

$$\varphi = \varphi_c + \frac{\omega L}{v}.$$

The described method was used for determining the heat-transfer coefficient in a close-packed stationary bed of spheres [2-5], in inserts for rotating regenerators [6], and in a fluidization bed [7]. The results, pertaining mainly to the bed of spheres, have revealed some advantages of the cyclic method: good enough reliability of test data covering the appropriate ranges of parameters $\alpha F/W$, H, and the feasibility of using it for other purposes too [8, 9].

An analysis of Eq. (13) shows (Fig. 3) that the cyclic method yields better results when $0 \leq H \leq 0.3$; when $H > 0.75$, the accuracy of the test data will, most probably, be unsatisfactory. The error increases also with higher values of the Biot number.

NOTATION

a	is the thermal diffusivity;
A_1, A_2	are the amplitude of the gas-temperature oscillations at the inlet and at the outlet, respectively;
$Bi = \alpha R/\lambda_M$	is the Biot number;
F	is the heat-transfer surface;
$H = \alpha F/W_M\omega$	is the dimensionless parameter;
L	is the length of package;
R	is the outside radius of cylinders;
r	is the radial coordinate;
t	is the temperature of gas;
t_M	is the temperature of solid;
t_{MO}	is the surface temperature of solid;
v	is the gas filtration velocity;
W	is the water equivalent of the gas;
W_M	is the water equivalent of the package;
$Z = (W/W_M\omega) \ln (A_1/A_2)$	is the dimensionless parameter;
α	is the heat-transfer coefficient;
λ_M	is the thermal conductivity of solid material;
τ	is the time;
φ_c	is the fundamental component of phase shift;
φ	is the total phase shift;
ω	is the radian frequency of temperature oscillation.

LITERATURE CITED

1. A. V. Lykov, Theory of Heat Conduction [in Russian], GITTL, Moscow (1952).
2. R. W. Dayton, S. L. Fawcett, R. E. Grimble, and C. E. Sealander, Batelle Memorial Inst., BMI-747 (1952).
3. R. M. G. Meek, Trans. ASME Proc. Confer. on Heat Transfer (1962).
4. V. Hlavačka, Zprava SVUSS, 67-05003 (1967).
5. H. Littman and D. E. Sliva, Heat Transfer, 7, CT 1.4 (1970).
6. V. Hlavačka, Candidate's Dissertation Prace, Prague (1968).
7. G. C. Lindauer, AIChEJ, 13, 1181 (1967).
8. H. Littman and R. G. Barile, Proc. AIChE Symp. on Fluidization, Dallas (1966).
9. G. A. Turner, AIChEJ, 13, 678 (1967).